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Generalized thermoelastic interaction in functional graded material with fractional order three-phase lag heat transfer

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Abstract: The present work is concerned with the solution of a problem on thermoelastic interactions in a functional graded material due to thermal shock in the context of the fractional order three-phase lag model. The governing equations of fractional order generalized thermoelasticity with three-phase lag model for functionally graded materials (FGM) (i.e., material with spatially varying material properties) are established. The analytical solution in the transform domain is obtained by using the eigenvalue approach. The inversion of Laplace transform is done numerically. The graphical results indicate that the fractional parameter has significant effects on all the physical quantities. Thus, we can consider the theory of fractional order generalized thermoelasticity an improvement on studying elastic materials.

Key words: three-phase lag model; functionally graded materials; fractional calculus; generalized thermoelasticity

1 Introduction

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. However, the theory does not apply to the behavior of many of the new synthetic materials of the elastomer and polymer type. In such cases, one needs to use a generalized thermoelasticity theory based on an anomalous heat conduction model involving time-fractional (non-integer order) derivatives. ABEL [1] who applied fractional calculus in the solution of an integral equation gave the first application of fractional derivatives. CAPUTO [2] gave the definition of fractional derivatives of order $0<\alpha\leq 1$ of continuous function. CAPUTO and MAINARDI [3−4] and CAPUTO [5] have employed the fractional order derivatives for the description of viscoelastic materials and have established the connection between fractional derivatives and the theory of linear viscoelasticity and found a good agreement with the experimental results. Among the few works devoted to applications of fractional calculus to thermoelasticity, we can refer to the works of POVSTENKO [6−7] who introduced a fractional heat conduction law and found the associated thermal stresses. SHERIEF et al [8], YOUSSEF [9] and EZZAT [10−11] introduced new models of thermoelasticity using a fractional heat conduction equation.

The generalization of the thermoelasticity theory is known as the dual-phase-lag thermoelasticity developed by TZOU [12] and CHANDRASEKHARIAH [13]. TZOU considered micro-structural effects in the delayed response in time in the macroscopic formulation by taking into account that increase of the lattice temperature is delayed due to photon-electron interactions on the macroscopic level. TZOU [12] introduced a two-phase-lag (2PHL) into both the heat flux vector and the temperature gradient. According to this model, classical Fourier's law $\boldsymbol{q}=-K\nabla T$ has been replaced by $\boldsymbol{q}(P,t+\tau_q)=-K\nabla T(P,t+\tau_T)$, where the temperature gradient ∇T at a point P of the material at time $t+\tau_T$ corresponds to the heat flux vector $\boldsymbol{q}$ at the same point in time $t+\tau_q$. Here K is the thermal conductivity of the material. The delay time τ_T is interpreted as that caused by the micro-structural interactions and is called the phase-lag of the temperature gradient. The other delay time τ_q is interpreted as the relaxation time due to the fast transient effects of thermal inertia and is called the phase-lag of the heat flux.

The generalization is known as three-phase-lag (3PHL) thermoelasticity, which is due to Roychoudhuri [14]. According to this model,

$$\boldsymbol{q}(P,t+\tau_q)=-[K\nabla T(P,t+\tau_T)+K^{*}\nabla\nu(P,t+\tau_\nu)]$$

where $\nabla\nu$ $(\dot{\nu}=T)$ is the thermal displacement gradient

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